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$$+2\log[\theta+\sqrt{(2+\theta^2)}]\}\Big]_{\theta=0}^{\theta=60}=3\sec\alpha\{\tfrac{1}{2}[60\pi\sqrt{(2+3600\pi^2)}$$
$$+2\log[60\pi+\sqrt{(2+3600\pi^2)}]-\tfrac{1}{2}[0\sqrt{2}+2\log(0+\sqrt{2})]\}$$
$$=3\sec\alpha\{30\pi\sqrt{(2+3600\pi^2)}+\log[60\pi+\sqrt{(2+3600\pi^2)}]-\log\sqrt{2}\}=4540 \text{ feet}.$$

DIOPHANTINE ANALYSIS.

116. Proposed by HARRY S. VANDIVER, Bala, Pa.

If n is an odd positive integer, and $1, n, n', n'', \ldots$ denote all its distinct divisors, then $2^n>2[n+1][n'+1][n''+1]\ldots$

Solution by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

There is a single exception $n=3$, for which $2^3=2[3+1]$. The corrected theorem may be proved by induction, using the following lemma:

If p is an odd prime number and d and π positive integers,

$$[1+d]^{p^\pi}>[1+d][1+pd][1+p^2d]\ldots[1+p^\pi d],$$

except for $d=1$, $\pi=1$, $p=3$, the equality sign then holding.

For proof we apply $p^\pi-1=p-1+p[p-1]+p^2[p-1]+\ldots+p^{\pi-1}[p-1]$.

$$\therefore[1+d]^{p^\pi}=[1+d][1+d]^{p-1}[1+d]^{p(p-1)}\ldots[1+d]^{p^{\pi-1}(p-1)}.$$

But $[1+d]^{t(p-1)}\gtreqless[1+td]^{p-1}\gtreqless 1+[p-1]td+[td]^{p-1}\gtreqless 1+[p-1]td+td$, if $p>2$. The equality signs hold simultaneously only when $t=d=1$, $p=3$. Hence, for $v>2$, $[1+d]^{t(p-1)}>1+ptd$ unless $t=d=1$, $p=3$, so that the lemma follows.

To prove the theorem by induction, we note that it is true for $n=p^\pi>3$, in view of the lemma for $d=1$. Assume that it has been verified for $n=p_1^{\pi_1}p_2^{\pi_2}\ldots$. We proceed to prove it true for $N=np^\pi$, p being prime to n. We have

$$2^N=[2^n]p^\pi\gtreqless[(1+1)(1+n)(1+n)\ldots]p^\pi$$
$$\gtreqless[(1+1)(1+p)\ldots(1+p^\pi)][(1+n)(1+pn)(1+p^2n)\ldots][(1+n')(1+pn')\ldots],$$

in view of the lemma. But the distinct divisors of N are

$$1, n, n', \ldots, p, pn, pn', \ldots, p^2, p^2n, \ldots, p^\pi, p^\pi n, \ldots$$

The theorem is therefore true for N.

118. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Find the two least integral numbers such that their sum shall be a square and the sum of their squares a biquadrate.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let x and y be the numbers, then for $x+y=1$, $x^2+y^2=13^4$,
$x=120$, $y=-119$.